Acquiring a Complete 3D Model from Specular Motion under the Illumination of Circular-Shaped Light Sources

Jiang Yu Zheng, Member, IEEE, and Akio Murata

Abstract—In this work, we recover 3D models of objects with specular surfaces. An object is rotated and its continuous images are taken. Circular-shaped light sources that generate conic rays are used to illuminate the rotating object in such a way that highlighted stripes can be observed on most of the specular surfaces. Surface shapes can be computed from the motions of highlights in the continuous images; either specular motion stereo or single specular trace mode can be used. When the lights are properly set, each point on the object can be highlighted during the rotation. The shape for each rotation plane is measured independently using its corresponding epipolar plane image. A 3D shape model is subsequently reconstructed by combining shapes at different rotation planes. Computing a shape is simple and requires only the motion of highlight on each rotation plane. The novelty of this paper is the complete modeling of a general type of specular objects that has not been accomplished before.

Index Terms—Shape from highlight, 3D model reconstruction, specular motion, epipolar-plane images, surface geometry, specular motion stereo.

1 INTRODUCTION

The current 3D shape reconstruction techniques in computer vision cannot deal with specular surfaces successfully. Even laser range finders are not able to get correct shape information at a surface with strong specular reflectance [13]. In the real world, however, many objects, from industrial products to daily goods, give off specular reflection. The objective of this work is to recover 3D shapes from specular motion by rotating objects so as to generate their graphics models. The constructed models are expected to be useful in CAD, industry prototyping, multimedia databases, information networks, and virtual reality.

Humans use two kinds of visual cues to perceive specularity. One is the intensity of a specularly reflected pattern, which differs from intensity of surface texture in the image. The other is the motion of a specularly reflected pattern on the surface, which also differs from the motion of surface texture when the viewpoint shifts. Studies on the specularity in vision have examined these two aspects. In the estimation of 3D points on a specular surface, some researchers have focused on the surface normal aspect in static images, e.g., Ikeuchi [1] and Healey [2]. Others have focused on the geometry of highlights. For example, Black et al. has studied the highlight disparity observed from two viewpoints [3]. Zisserman et al. has studied the local motion aspect of the highlight under a point light [4]. When the camera is moved, a surface curve on the object could be measured from a highlight point [12], [14]. However, these efforts were mainly aimed at distinguishing highlight points from texture points. The curves recovered under a point light illumination were not sufficient to generate object models in real applications. One of our goals is then to build complete models of specular objects.

The approach employed here is to move (rotate) an object under properly set fixed illuminations [9], [11], [16]. The lights are extended shapes of illuminations that allow stripes of highlights to be observed on the object surfaces. With the motion of highlight stripes on a surface, the surface’s shape can be recovered. By using epipolar plane images (EPIs), the motions of highlights shifting on different surface shapes can be clearly displayed, and qualitative highlight motion characteristics can be explored [11], [16]. Light strip sources have been used to produce “planar rays” model of illumination for object recovery [9], [11], [16].

In this paper, we use circular-shaped light sources to illuminate rotating objects in order to obtain a more complete model, as well as a simple and robust algorithm. Every object point, regardless of its normal surface, is highlighted once during the object rotation if there is no light occlusion. This technique allows us to recover more specular surfaces than those recovered by using the light strips. Moreover, it simplifies the geometry relation between the normal, light, and camera so that the shape computation can be carried out independently on each epipolar plane image. This is also much more effective than using light strips in the model acquisition.

The new illumination allows us to reconstruct objects with smooth surfaces, particularly small objects. Previously, laser scanning could not accurately measure such small objects because the width of the laser stripe was not narrow enough. The accuracy of our method is high in principle if we zoom in the object. This is because the width of a highlighted stripe is related to the surface shape. Our use of epipolar plane images other than the original input images yields accurate highlighted positions because we can locate highlights spatially and temporally in the EPIs, and this directly improves the estimation of surface shapes. We show that the displayed highlight trace is sensitive to small changes in the surface shape.

In the next section, we will clarify previous work regarding shape, light, and complexity of the algorithm, which has not been precisely formalized before. In Section 3, we derive the ideal light source. In Sections 4 and 5, we will show that the 3D estimation is simpler than the previous work under both double and single light illuminations. Finally, we will give new experimental results that have not been obtained before.

2 FORMALIZING PREVIOUS WORKS USING PLANAR RAYS

Under orthogonal projection, the newest results on shape estimation of specular objects are from acquiring 3D surface and model performed on rotating objects [9], [11]. For a rotating object, the camera axis is set orthogonal to the rotation axis (Fig. 1) and a
sequence of images is taken. We obtain a sequence of discrete images in relation to the rotation angle. In order to illuminate a specular surface, extended shape of lights were used to produce highlights on the object’s surface. Either a linear or curved light source, or a combination of light strip sources is set around the object in a plane that contains the rotation axis. These lights are located far away from the object compared to the object size so that the rays from a light point to different object points have the same direction. Rather than shapes of lights, it is the composition of rays, called plane of rays, that determines the geometry for 3D estimation.

Having the next two kinds of highlight information helps us learn surface shapes: one is the observed shape of highlight stripe in the image and the other is its motion on the surface due to the object rotation. When the highlighted stripe moves over a surface during rotation, the surface shape can be determined from these two types of information, which can be parameterized separately according to the normal direction of stripe in the images and trajectories of highlights in the image sequence.

Actual computations involve epipolar plane images (EPI). As the object rotates, highlights gradually move over the surface. Three-dimensional positions of highlight-passed points are computed on each rotation plane by using the moving trajectories of highlights in the EPI and the corresponding highlight directions (or gradient directions of highlighted stripes) in the images [16].

With one or two planes of rays in various directions from the camera axis, we summarize our algorithms (Table 1) for different types of shapes. If planar rays are set from a single direction, the shape recovery is a first-order differential equation. On the other hand, setting multiple planes of rays will make the problem as simple as a linear equation. Both cases need the gradient of the highlight stripe in the images. If the object is a cylinder or the planar rays are aligned with the camera direction, the extraction of the highlight normal in the image is unnecessary. It is possible for all of the surface points to be highlighted, but only when planar rays are with the camera (\( \phi = 0 \)). Otherwise, a surface point whose normal direction is close to the rotation axis might not be highlighted in the rotation period. This is one reason why we need to search for a new type of illumination.

### Table 1

<table>
<thead>
<tr>
<th>Shape Recovery under Different Conditions such as Object Shape, Number of Planar Rays, and Locations of Rays</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder ((n = 0))</td>
</tr>
<tr>
<td>General types of objects</td>
</tr>
<tr>
<td>Others</td>
</tr>
</tbody>
</table>

\( n_y \) is surface normal component along rotation axis.

---

**3 Determining the Ideal Light Shape**

What is the ideal illumination? A point light can only recover a surface curve during rotation. Generating a complete model requires more highlighted points in its rotation. We want to achieve the following: 1) every surface point on the surface should be highlighted by the illumination, 2) the computation should be simple and robust under the light and system configuration. We will show that circular lights not only satisfy the requirements but also eliminate the computation of highlight shape in the images required in the light strip case. The shape recovery, therefore, is done on each EPI separately, which is more stable and efficient.

Let us first look at the system settings. We choose the axis of rotation as the Y-axis of object coordinate system O-XYZ and the y-axis of camera coordinate system C-xyz. The rotation angle is known and the rotation is clockwise in the rotation plane. As an
object rotates, the spatial-temporal volume is increased from the continuous images. A surface point $P(X, Y, Z)$ in the system O-XYZ is projected to $p(x, y)$ in the image frame and is mapped to $(x(\theta), y(\theta), \theta)$ in the spatial-temporal volume. Thus, a continuous trajectory $p(x(\theta), y(\theta), \theta)$ is formed in the volume when point $P$ rotates around. Note that both O-XYZ and C-xyz systems are left-handed systems. The surface normal at the point is denoted as $N = (n_x, n_y, n_z)$ in the system O-XYZ. The component $n_x$ along the rotation axis is constant during the rotation and the component $(n_x, n_y, n_z)$ in the rotation plane faces all orientations periodically. A ray is from a point of light to the object, but we denote its opposite direction as $L = (l_x, l_y, l_z)$ in the system C-xyz.

By analyzing $L$, we will select an extended illuminant that makes every surface point highlighted when its normal component $N$ lies in the rotation plane. A ray is from a point of light to the object, but we denote its opposite direction as $L = (l_x, l_y, l_z)$ in the system C-xyz.

Let us figure out the shape of the illuminant using a Gaussian Sphere in the system C-xyz (depicted in Fig. 2). The surface normal $N$ of a point is displayed on the sphere without its real position on the object. The camera is located at the inverse side of the rotation axis (y axis). The details of the proof are omitted due to the limited space. Since $L$ is also a unit vector on a sphere, it is now on the intersection of the sphere and the plane, which subsequently creates a circle $C$ that is symmetric with respect to the xz-plane (see Fig. 2). The rays $L$ from the light should pass through circle $C$ and reach the rotation center. Therefore, the rays are symmetric with respect to an axis of symmetry that is parallel to the xz-plane. They compose a cone called the cone of rays.

A light set capable of generating such rays can have various shapes, but it should be on the cone of rays. The simplest one is a light source that forms an ellipse on the cone of rays. Fortunately, we can use a circular-shaped light source to achieve such a relatively simple illumination. The circular-shaped light should be set so that 1) it is symmetrical with respect to the xz plane, 2) all of the points on the light are at the same distance from the rotation center (origin of O-XYZ), and 3) one light point is on the camera axis behind the object. In a real setting, we look at the two intersecting points of the light circle on the xz plane. According to Fig. 2, one midpoint is located on the camera axis (in the z direction) and the other has an orientation $\phi$ from the camera axis (in the direction of $L_0$). A large light compared with the object size is preferred because its rays to the object will be close to the rotation plane, say $\phi/2 \in (-\pi/2, \pi/2)$. In other words, the surface normal $N$ should be highlighted in the vertical plane YON$_0$ where $N \cdot VON_0 = \phi/2$.

What sort of ray should be lit for such a normal? According to the specular reflection criterion, the ray highlighting the point must be in plane VON and must have an angle of incident $\angle LON$ equal to the angle of reflection $\angle NOV$. For all possible $N_0$, we want to see the distribution of the ray orientations. If $N$ varies from $-\pi$ to $+\pi$ in plane YON, plane VON rotates around axis $z$. We can deduce that ray $L$ on the extension of plane VON will make a circle on the Gaussian Sphere. The circle is symmetric with respect to the horizontal plane. It goes through vector $z$ and vector $L_0(\angle L_0OV = \phi)$ in the horizontal plane. Further, a brief deduction can show that the direction of light $L(l_x, l_y, l_z)$ satisfies

$$l_z = \tan(\phi/2)(1-l_z),$$

which means that vector $L$ is in a (vertical) plane parallel to the rotation axis (y axis). The details of the proof are omitted due to the limited space. Since $L$ is also a unit vector on a sphere, it is now on the intersection of the sphere and the plane, which subsequently creates a circle $C$ that is symmetric with respect to the xz-plane (see Fig. 2). The rays $L$ from the light should pass through circle $C$ and reach the rotation center. Therefore, the rays are symmetric with respect to an axis of symmetry that is parallel to the xz-plane. They compose a cone called the cone of rays.
ideal rays centralized at the origin. Given a circular-shaped light source with known diameter $\gamma$, and selecting an orientation $\phi/2$ for observing the highlight, the light source can be set uniquely as follows: Two mid-points will a) have distance $\gamma/(2\sin(\phi/2))$ from the rotation axis and b) at the orientations $\phi$ and $\pi$ from the camera axis. Their coordinates are $(\gamma\cos(\phi/2), 0, -\gamma\cos(\phi/2)\cot\phi)$ and $(0, 0, \gamma/(2\sin(\phi/2)))$.

4 SPECULAR MOTION STEREO

A circular-shaped light source guarantees that a surface point will be highlighted by a certain part of the light source when the normal of the point rotates to the given orientation $\phi/2$. From the camera geometry (Fig. 3), we know that the viewing direction under the orthogonal projection is $V(-\sin\theta, 0, \cos\theta)$ in the system O-XYZ.

The image position of a surface point viewed at the rotation angle $\theta$ is written as

$$x(\theta) = P \cdot x = X(\theta)\cos + Z(\theta)\sin\theta,$$

where $x$ is the unit vector of the horizontal image axis. Obviously, the position that reflects the light to the camera will shift on the surface during rotation; surface points in turn pass the direction $\phi/2$.

If we set two circular-shaped light sources to pass through $V$ and in the directions $\phi_1$ and $\phi_2$ from the camera axis (Fig. 4), the arbitrary point will reflect them to the camera as its normal rotates to the two orientations. The normal is in turn in the vertical planes that have angles $\phi_1/2$ and $\phi_2/2$ from the camera axis. The point has a delay $\Delta\phi = \phi_1/2 - \phi_2/2$ in reflecting the two lights. Two lines of sight through the projected highlights geometrically cross at the surface point and this determines the point’s position. This can be seen in the following. The equations for lines of sight at the two orientations are written as

$$x_1(\theta) = X(\theta)\cos\theta + Z(\theta)\sin\theta,$$

$$x_2(\theta + \Delta\phi) = X(\theta)\cos(\theta + \Delta\phi) + Z(\theta)\sin(\theta + \Delta\phi),$$

which yield the position of the surface point as

$$\begin{pmatrix} X(\theta) \\ Z(\theta) \end{pmatrix} = \frac{1}{\sin\Delta\phi} \begin{pmatrix} \sin(\theta + \Delta\phi) & -\sin\theta \\ -\cos(\theta + \Delta\phi) & \cos\theta \end{pmatrix} \begin{pmatrix} x_1(\theta) \\ x_2(\theta + \Delta\phi) \end{pmatrix},$$

where $X(\theta)$ and $Z(\theta)$ are coordinates of the surface point at the rotation plane. The principle used here is similar to the principle of motion stereo, except that the focused points are highlights other than edges from the texture or corners. We call it specular motion stereo. The computation is very simple and the result is only related to the light settings, rotation angle, and image positions of the highlights.

The question now is how to locate the two highlights that have a delay in the rotation angle. In an epipolar plane image, a surface point has its moving trajectory as a sinusoidal curve based on (2), even if the trajectory cannot be visually located in the images when the shape is smooth and lacks texture. In principle, we need at least two projections of the point on the trajectory to determine the 3D position. Two highlighted positions provide the information. When a highlight shifts on the object surface, a bright trace is observed in the EPI (Fig. 5). For two light sources, the sinusoidal trajectory of the surface point crosses two highlight traces with the fixed delay. We, therefore, track the first highlight trace and, for each point on the trace, we find its correspondence at the second highlight trace, which has delay $\Delta\phi$ from the first one.

The qualitative characters of the highlight traces for different surfaces in this paper are the same as those described in [9], [11], [16] and are omitted here due to the limited space. Any zero curvature point (appearing on the planes) on the rotation plane will yield two horizontal segments of highlight traces in the EPI; they will have delay $\Delta\phi$ in reflecting two lights. By measuring the difference in $\theta$ between the two segments, we can calibrate the orientations of two light sources.

5 SHAPE ESTIMATION UNDER SINGLE SET OF CONIC RAYS

With one circular-shaped light source, surface points with normals of any type can be computed by solving a first-order differential equation; this has been done with a more complicated form for the planar rays illumination (Table 1). We will see a simplified result under the conic rays illumination. As a highlight shifts over the object surface, the line of sight also moves. Such a move can be described by taking a derivative of the equation of the line of sight, which is shown as (2),

Fig. 5. Tracking highlight traces in an EPI and matching correspondences of a surface point at highlight traces.

Fig. 6. Test of our algorithm on a sphere (5cm diameter). (a) Original image. (b) Recovered model.
where $x'(\theta)$ is the direction of highlight trace in the EPI. The $(X', Z')$ denotes the move of the highlight on the object surface in the coordinate system O-XYZ. The move $(X', Z')$ determines the tangent of the shape in the rotation plane. Because the tangent direction should be perpendicular to the normal direction in the rotation plane at any time, the normal then is simply

$$
\frac{\partial X}{\partial \theta} \cos \theta + \frac{\partial Z}{\partial \theta} \sin \theta - X \sin \theta + Z \cos \theta = 0.
$$

Combining (2), (6), and (7), we obtain two differential equations separately in the forms of $X$ and $Z$. 

Fig. 7. A bottle made of metal and its recovered 3D model. (a) An original image. (b) Recovered whole model. (c) Upper part of object as a wire frame model.

Fig. 8. Recovered model of a metallic object. (a) An original image. (b) An EPI on which highlight traces can be observed. (c-d) Recovered model.
in the domains $\theta \neq \pi/2$ and $\theta \neq 3\pi/2$, and

$$\frac{\partial X}{\partial \theta} \cos \frac{\phi}{2} - X \frac{\cos(\theta + \frac{\phi}{2})}{\sin \theta} =$$

$$= \frac{\partial x}{\partial \theta} \cos \frac{\phi}{2} - x \cot \theta \cos \left(\theta + \frac{\phi}{2}\right)$$

in the domains $\theta \neq 0$ and $\theta \neq \pi$. Two equations are generated because of the singularity of cosec and sec functions in the equations at the particular angles $\theta = 0$ and $\theta = \pi$. Their solutions are written as

$$Z(\theta) =$$

$$= \left( \int_{\theta_0}^{\theta} \sin \frac{\phi}{2} \left( x'(\lambda) + x(\lambda) \tan \lambda \right) e^{\int_{\theta_0}^{\lambda} \frac{\sin(\phi/2)}{\cos^{3/2} \lambda} d\lambda} \right)$$

$$e^{-\int_{\theta_0}^{\theta} \frac{\sin(\phi/2)}{\cos^{3/2} \lambda} d\lambda}$$

in the domain $\theta \neq \pi/2$ and $\theta \neq 3\pi/2$

$$X(\theta) =$$

$$= \left( \int_{\theta_0}^{\theta} \cos \frac{\phi}{2} \left( x'(\lambda) - x(\lambda) \cot \lambda \right) e^{\int_{\theta_0}^{\lambda} \frac{\sin(\phi/2)}{\cos^{3/2} \lambda} d\lambda} \right)$$

$$e^{\int_{\theta_0}^{\theta} \frac{\sin(\phi/2)}{\cos^{3/2} \lambda} d\lambda}$$

in the domain $\theta \neq 0$ and $\theta \neq \pi$. We compute the solutions by using their numerical expansions to determine X or Z and then the corresponding X and Z using (2). By tracking the highlight move that is a trace in the EPI, we can estimate the object shape from the image position of highlight $x(\theta)$ and rotation angle $\theta$. This computation is an integral from an initial 3D position $(X_{i0}, Z_{i0})$ known on the object. The integral calculation has been achieved in a more complicated case [16], where even the derivative terms of the highlight position $x'(\theta)$ are excluded. The known initial position is then obtained at fixed points from an edge of surface pattern or a corner. The 3D position of a fixed point is estimated using shape from motion algorithm.

6 EXPERIMENTAL RESULTS

We have performed experiments on real objects with smooth surfaces. These objects had metallic or plastic surfaces. Fig. 4 shows the system setting for modeling the objects. Two circular fluorescent lights (commercial product) with diameters of 32cm
and 38cm were used. The image projection of the rotation axis was determined by imaging a standing pole at the center of a turn-table. The accurate light direction was then calibrated using an EPI taken from a vertical planar mirror at the center of the turn-table. The camera was about 2.5m away from the object and was equipped with a lens having a long focal length. Rotation degree and number of EPIs used are determined from required patch density of generated model. Our system takes images at every one degree of rotation. In principle, every point is highlighted once in a complete rotation period under a circular-shaped light source. However, if an object has a very deep concave shape, the ray may be occluded so that no highlight is observable on the surface. The major experiments used two cones of rays. Fig. 6 gives an example of a specular sphere under illumination of conic rays. The shaded area briefly indicates the scale of our experimented objects.

The method proposed here will work on metallic surfaces where specular reflectance is the major component of reflectance as well as on plastic surfaces where the diffused component of reflectance is large. Fig. 9 shows a plastic toy dog and its recovered model. Although the specular reflection component is not strong, we could still track highlights and recover major parts of the object. Fig. 9b shows an EPI of the object, in which we can notice a difference in the highlight trace compared to the metallic one.

We matched highlight traces from two illuminations and computed 3D locations of points on the traces. Parts on the model are missing because of the lack of clear highlights on weak specular surfaces, ray occlusion, and failures in highlight extraction. It is interesting to note that the unevenness of a surface composed of small planes can be observed in an epipolar plane image; it appears as a zigzag highlight trace. This unevenness is hard to distinguish in the original image, since the width of the highlight tells us nothing about the surface shape if we do not know the size of the light. In epipolar plane images, however, such small planes appear as horizontal traces and any unevenness of a surface makes these traces zigzag. Fig. 10 shows such an example.

The error of estimated position is from the inaccurate system setting, feature extraction, and approximation of the illumination model. The accuracy in the system setting and feature extraction can be improved by a careful calibration and refining algorithms. The conic ray model requires that the circular lights have enough large size compared with objects to be measured. We have experimented that if the object size increases to 40 percent of the diameter of circular light, the relative shape error may reach 13 percent in the xz plane and increase to 30 percent at the farthest position from the xz plane (Fig. 11).

The computation algorithms and object types can be unified with the illumination of conic rays proposed in this paper. Table 2 summarizes the algorithms used in the illumination of conic rays. The computation can be carried out on each EPI to achieve better performance. The light around an object from the camera orientation in Table 1 is one of the cases where a circular-shaped light source is set at the direction $\phi = 0$. The conic rays degenerate to planar rays.

7 CONCLUSION

In this paper, we reconstruct 3D models of objects with specular surface reflections. Such reconstruction has been an unsolved problem in 3D object modeling using images. We generalized methods of shape recovery that use circular-shaped light sources to illuminate rotating objects. This allows every point on an object to be highlighted at a desired orientation regardless of its surface normal. Additionally, a computation on general types of objects becomes very simple and is carried out on each rotation plane. The light direction is relatively unconstrained compared with a previous method of using light strip sources. We have also performed experiments on real objects with metallic and plastic surfaces and obtained good results.

<table>
<thead>
<tr>
<th>Conic Rays (circular lights)</th>
<th>Multiple Lights Mode</th>
<th>Single Light Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>General types</td>
<td>Linear equations</td>
<td>1st-order differential equation &amp; Fixed points</td>
</tr>
</tbody>
</table>

Fig. 11. Relative error in the estimated shape due to the approximation of ideal conic rays. The shaded area briefly indicates the scale of our experimented objects.
REFERENCES


